# College Algebra

Chapter 1

# Note:

This review is composed of questions similar to those from the chapter review at the end of chapter 1. This review is meant to highlight basic concepts from chapter 1.  It does not cover all concepts presented by your instructor. Refer back to your notes, unit objectives, handouts, etc. to further prepare for your exam.

This review is available in alternate formats upon request.

Find domain of $f(x) = \sqrt{2 - x}$ and the following: f(x-2), -f(x)

Domain: Set what's inside the square root to be greater or equal to zero and solve for x

$$2 - x \geq 0$$
$$2 \geq x$$
or
$$x \leq 2$$

This means that anything more than 2 would cause what's under the square root to be a negative which indicates imaginary numbers. The domain can only consist of real numbers so x must be less than 2. The domain is then $\{x|x \leq 2\}$

f(x-2): Set x=x-2 and simplify

$$f(x-2) = \sqrt{2-(x-2)}$$
$$f(x-2) = \sqrt{2-x+2}$$
$$f(x-2) = \sqrt{4-x}$$

-f(x): Multiply both sides of the function by -1

$$-1(f(x)) = -1(\sqrt{2-x})$$
$$-f(x) = -\sqrt{2-x}$$

Find $f+g,\ f-g,\ f\cdot g,\ \frac{f}{g}$ for the following pair of functions. State the domain of each of these functions.

$$f(x)=3x^2+x+1; g(x)=3x$$

$f+g$: Add the functions and combine like terms
$(f+g)(x)=3x^2+x+1+3\text{x}$
$(f+g)(x)=3x^2+4x+1$

$f-g$: Subtract the functions and combine like terms
$(f-g)(x)=3x^2+x+1-3\text{x}$
$(f-g)(x)=3x^2-2x+1$

$f\cdot g$: Multiply the functions
$(fg)(x)=(3x^2+x+1)(3\text{x})$
$(fg)(x)=9x^3+3x^2+3x$

$\frac{f}{g}$: Divide the functions
$\left(\frac{f}{g}\right)(x)=\frac{3x^2+x+1}{3x}$ (cant be simplified any more)

Domain: Find the domain of f and g first.
Since both are functions that do not contain functions over functions, variables inside roots, fractional exponents, or negative exponents, the domain for f and g is all real numbers.
This is also the domain for $f+g, f-g,\ and\ f\cdot g$.
To find the domain of $\left(\frac{f}{g}\right)(x)$, we need consider the domains of f and g but also that $g(x)\neq 0$.
If x=0, then g(x)=0 which is not allowed. So the domain for $\left(\frac{f}{g}\right)(x)$ is $x\neq 0$.

Find the difference quotient of $f(x) = -2x^2 + x + 1$; that is, find $\frac{f(x+h)-f(x)}{h}, h \neq 0$

$$\frac{-2(x+h)^2 + (x+h) + 1 - (-2x^2 + x + 1)}{h}$$

$$\frac{-2(x+h)(x+h) + (x+h) + 1 - (-2x^2 + x + 1)}{h}$$

Multiply (x+h)(x+h)

$$\frac{-2(x^2 + 2xh + h^2) + (x+h) + 1 - (-2x^2 + x + 1)}{h}$$

Distribute -2

$$\frac{-2x^2 - 4xh - 2h^2 + x + h + 1 - (-2x^2 + x + 1)}{h}$$

Distribute -1 in front of $(-2x^2 + x + 1)$

$$\frac{-2x^2 - 4xh - 2h^2 + x + h + 1 + 2x^2 - x - 1}{h}$$

$$\frac{-4xh - 2h^2 + h}{h}$$

Factor out an h

$$\frac{h(-4x - 2h + 1)}{h}$$

Cross out h

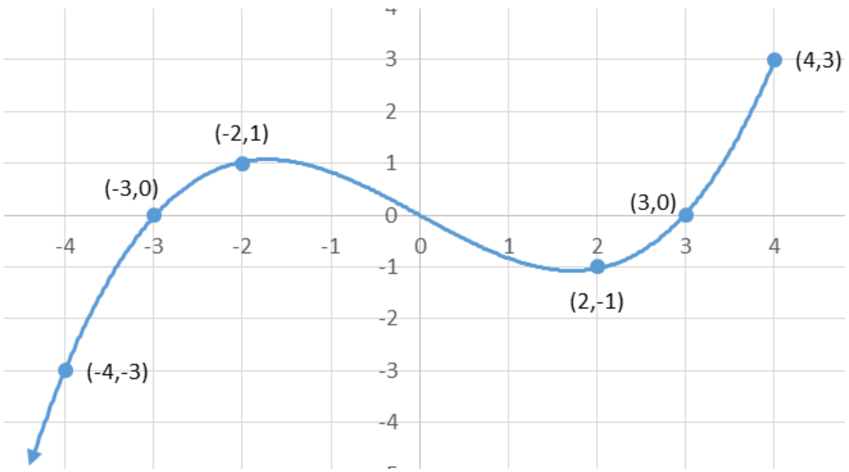
$$-4x - 2h + 1$$

Difference quotient is $-4x - 2h + 1$

Use the graph of the of function f shown to find:

a) The domain and range

b) The intervals on which f is increasing or decreasing




a) The domain is the set of possible x-values. The smallest (most negative) x-value is $-\infty$ and the largest is 4. Thus the domain is $(-\infty, 4)$.

The range is the set of possible y-values. The smallest (most negative) y-value is $-\infty$ and the largest is 3. Thus the domain is $(-\infty, 3)$.

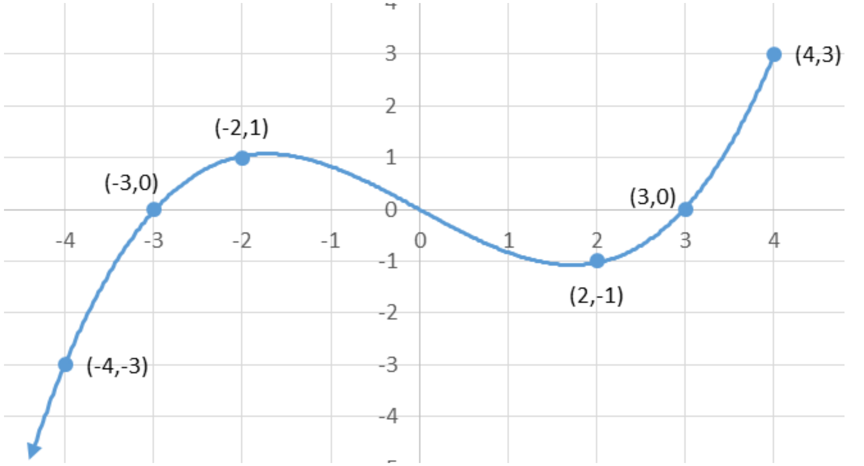
b) As we move left to right, if we move up, we are increasing and as we move down, we are decreasing. Starting at the left and move right, we are going up (increasing) until x=-2. Then the we start to move down (decreasing) until 2 where we move up.

The function is increasing $(-\infty, -2) \cup (2, \infty)$ and decreasing $(-2, 2)$.

(Remember the point where it changes directions is not included)

Use the graph of the of function f shown to find:

a) The local min and max
b) The absolute min and max
c) Whether the graph is symmetric to x-axis, the y-axis, or origin
d) Whether the function is even, odd, or neither
e) Find the intercepts




a) The local minimums are low points: local min is -1 occurs at x=2
local maximums are the high points: local max is 1 occurs at x=-1
Although the point (4,3) is a high point, there is not a lower point on both sides of it. This is a condition of local mins and maxes but for absolutes

b) The absolute min is the lowest point of all (unless its an open interval end point): None
The absolute max is the highest point of all (unless its an open interval end point): absolute max is 3 and occurs at x=4

c) Symmetry about the x-axis: No because the graph is not mirrored across the x-axis and every point (x,y), the point (x,-y) is not on the graph
Symmetry about the y-axis: No because the graph is not mirrored across the y-axis and every point (x,y), the point (-x,y) is not on the graph
Symmetry about the orgin: No because every point (x,y), the point (-x,-y) is not on the graph. If the graph continued on after x=4, then we would have had this symmetry.

d) Neither because it is not symmetric about the y-axis and the origin.

e) x-intercepts: -3,0,3 and y-intercept:0

Determine algebraically whether the functions are even, odd, or neither

$$f(x) = \frac{4+x^2}{1+x^4} \qquad g(x) = \frac{3x}{4+x^6}$$

**f(x):** Replace x with –x and simplify

$$f(-x) = \frac{4+(-x)^2}{1+(-x)^4}$$

$$f(-x) = \frac{4+x^2}{1+x^4}$$

This is equivalent to $f(x) = \frac{4+x^2}{1+x^4}$ so the equation is even

**g(x):** Replace x with –x and simplify

$$g(-x) = \frac{3(-x)}{4+(-x)^6}$$

$$g(-x) = \frac{-3x}{4+x^6}$$

$$g(-x) = -\frac{3x}{4+x^6}$$

This is equivalent to –g(x) so the equation is odd.

Find the average rate of change from 2 to 3 for the following function

$$f(x) = 2 - 5x$$

Average rate of change means finding the slope

Remember slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

We have $x_1 = 2$ and $x_2 = 3$. To find the y-values, we plug the x-values into the given function.

$$y_1 = f(2) = 2 - 5(2) = 2 - 10$$
$$y_1 = -8$$

$$y_2 = f(3) = 2 - 5(3) = 2 - 15$$
$$y_2 = -13$$

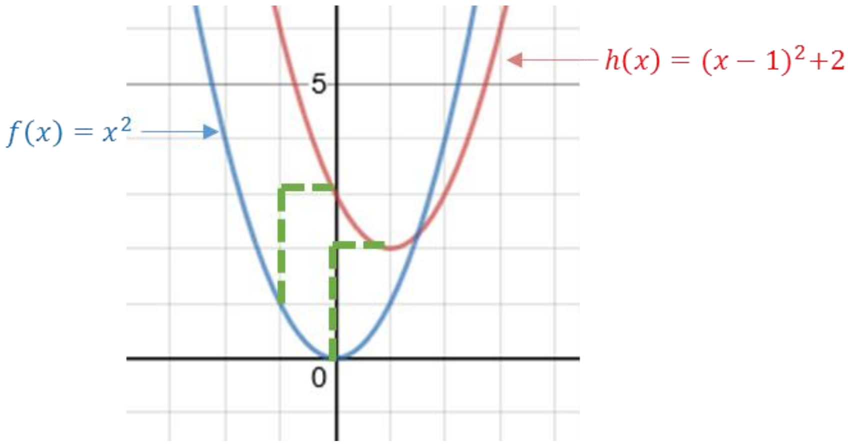
We now can plug the values into the slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-13 - (-8)}{3 - 2} = \frac{-5}{1} = -5$$

Thus the average rate of change from 2 to 3 is -5.

Graph $h(x) = (x-1)^2+2$ using the techniques of shifting, compressing or stretching, and reflections. State the domain and based on the graph, find the range.

The parent graph would be $f(x) = x^2$ (shown in blue)
$h(x)$ is obtained by shifting the graph to the right by 1 and up by 2.
There isn't any compressing, stretching, or reflections.




The domain is all real numbers and the range is $[2, \infty)$

For the piece-wise function

a) Find the domain

b) Graph the function

c) Find where f(x)=0

d) Is f continuous on its domain?

$$f(x) = \begin{cases} 3x & if\ -2 \le x \le 1 \\ x+1 & if\ x > 1 \end{cases}$$

a) The domain (the possible x-values) are found on the right side of the equation.

$-2 \le x \le 1$ and $x > 1$

From first inequality we see that -2 is the smallest number and from the second inequality we see that ∞ is the largest number.

The domain is $[-2, \infty)$ or $\{x | x \ge -2\}$

b) Set each part of the equation equal to zero and solve

*Top half:*

$3x = 0$

$x = 0$

Since 0 is in the domain of the top portion ($-2 \le x \le 1$ ), f(0)=0

*Bottom half:*

$x + 1 = 0$

$x = -1$

x=-1 is not in the domain of the bottom portion ($x > 1$). So (-1,0) is not an answer to where f(x)=0.

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c) To graph the function, we being by putting the endpoints into the equation and noting if the point is included or not.

*Top half*

i) x=-2 $f(-2) = 3(-2) = -6$ The point (-2,-6) is included (because of the ≤). Plot this point as a filled-in circle.

ii) x=1 $f(1) = 3(1) = 3$ The point (1,3) is included as well. Plot this point as a filled-in circle

iii) connect with the previous points.

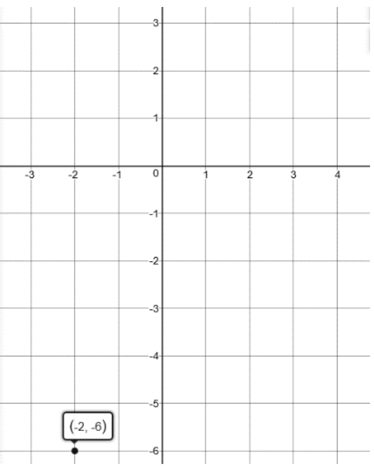
*Bottom half*

iv) x=1 f(1)=1-1=0 The point (1,0) is not included. Plot this point as an open circle.

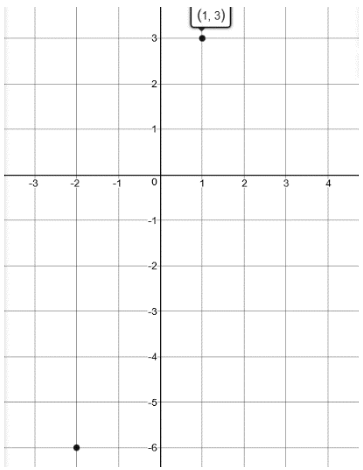
v) While we can't plot the right hand endpoint (∞), we can plot another point that will help make the line.

Let x=4 f(4)=4-1=3 Plot the point (4,3) as a small dot. Starting at the previous point draw a line that goes through (5,6)

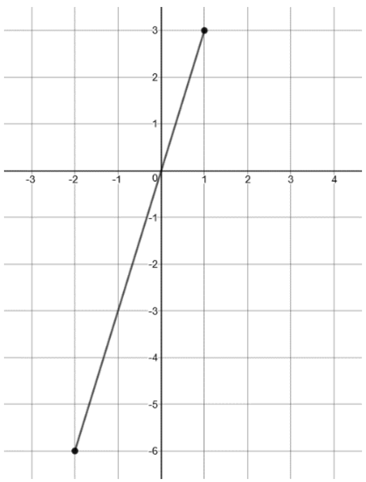
i)



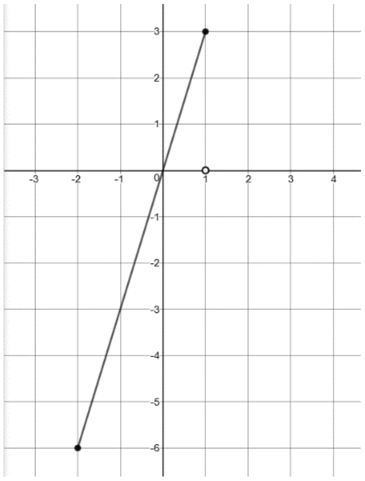

ii)



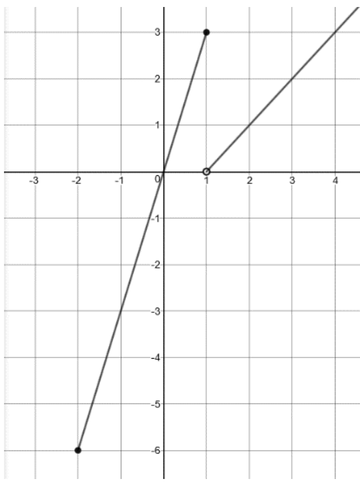

iii)



iv)



v)



d) When we trace the graph from left to right we would need to lift our hand/pencil at x=1 to go from one portion to another, so f is not continuous on its domain.

p is inversely proportional to b. If p= 15 when b is 4, what is p when b is 19. Round to 2 decimal places.

1) Start by writing the equation using k for the constant of proportionality.

p is $\underbrace{\text{inversely proportional}}$ to b.

$$p = \frac{k}{b}$$

$$p = \frac{k}{b}$$

2) Find k using the known b and its corresponding p.

$$15 = \frac{k}{4}$$

Multiply both sides by 4

$$15 \cdot 4 = k$$

$$60 = k$$

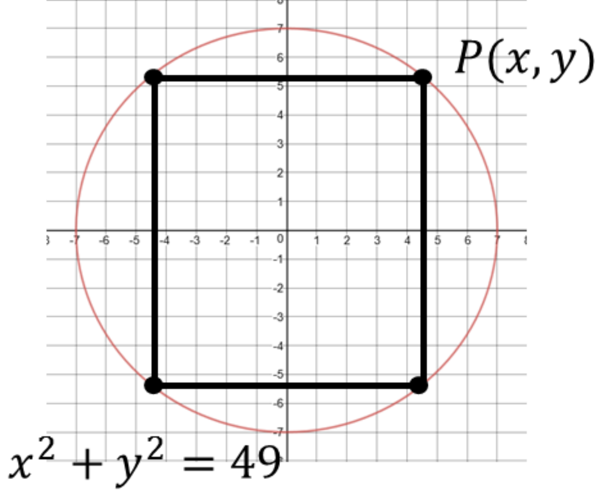
3) Using the k=60, put in the b=19 and the k into the equation to solve for p.

$$p = \frac{60}{19} \approx 3.16$$

A rectangle is inscribed in a circle of radius 7 (see image). Let P(x,y) be the point in quadrant I that is a vertex of the rectangle and is on the circle.

a) Express the area A of the rectangle as a function of x.

b) Express the perimeter p of the rectangle as a function of x.




a) Remember that area is length times width.

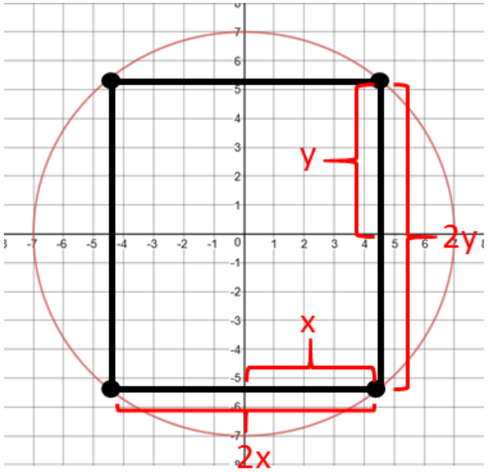
The width can be expressed as 2x and the length is 2y (see image for details)

Area=$2x\cdot 2y=4xy$

Since we want the area function to be function of x, we can solve the circle's equation for y and substitute this into the y in 4xy.

$$x^2+y^2=49$$
$$y^2=49-x^2$$
$$y=\sqrt{49-x^2}$$

Area function is $A(x)=4x\sqrt{49-x^2}$




b) Perimeter of a rectangle is twice the length plus twice the width. Again, the width can be expressed as 2x and the length is 2y.

Perimeter =$2(2x)+2(2y)=4x+4y$

Since we want the perimeter function to be function of x, we can solve the circle's equation for y and substitute this in for y. From above, we found $y=\sqrt{49-x^2}$. So $p(x)=4x+4\sqrt{49-x^2}$

If a baseball falls from a height of 200 feet, the height H (in feet) after x seconds is $H(x) = 200 - 16x^2$ .

Round the answers to the following questions to 2 decimals places.

a) How long does the baseball take to reach 100 feet?

b) How long does the baseball take to reach the ground?

a) How long does the baseball take to reach 100 feet?

$$100 = 200 - 16x^2$$

Subtract 200 from both sides

$$-100 = -16x^2$$

Divide both sides by -16

$$6.25 = x^2$$

Take the square root of both sides (ignore the negative value from the square root)

$$2.5 = x$$

After 2.5 seconds, the ball is 100 feet.

b) How long does the baseball take to reach the ground?

The ground has a height of 0 feet so set H(x)=0 and solve for x

$$0 = 200 - 16x^2$$

Subtract 200 from both sides

$$-200 = -16x^2$$

Divide both sides by -16

$$12.5 = x^2$$

Take the square root of both sides

$$3.54 = x$$

The ball hits the ground at 3.54 seconds.