College Algebra

Chapter 7

Note:

This review is composed of questions similar to those in the chapter review at the end of chapter 6. This review is meant to highlight basic concepts from chapter 6. It does not cover all concepts presented by your instructor. Refer back to your notes, unit objectives, handouts, etc. to further prepare for your exam.

Write the first five terms

$$\{a_n\} = \left\{\frac{3+2n}{4n}\right\}$$

$$a_1 = \frac{3+2(1)}{4(1)} = \frac{5}{4}$$

$$a_2 = \frac{3+2(2)}{4(2)} = \frac{7}{8}$$

$$a_3 = \frac{3+2(3)}{4(3)} = \frac{9}{12} = \frac{3}{4}$$

$$a_4 = \frac{3+2(4)}{4(4)} = \frac{11}{16}$$

$$a_5 = \frac{3+2(5)}{4(5)} = \frac{13}{20}$$

$$a_1 = 6 \quad a_n = 1 + 2a_{n-1}$$

$$a_1 = 6$$

$$a_2 = 1 + 2(6) = 13$$

$$a_3 = 1 + 2(13) = 27$$

$$a_4 = 1 + 2(27) = 55$$

$$a_5 = 1 + 2(55) = 111$$

Determine if the sequence is arithmetic, geometric, or neither.
If arithmetic, find the common difference and the sum of the first n terms
If geometric, find the common ration and the sum of the first n terms.

4, -8, 16, -32…

Geometric
No common difference
Common Ratio: $\frac{-8}{4} = -2;\ \frac{16}{-8} = -2, \frac{-32}{16} = -2$
r=-2

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right) = 4\left(\frac{1-(-2)^n}{1-(-2)}\right)$$
$$= 4\left(\frac{1-(-2)^n}{3}\right)$$

3, 6, 9, 12…

Arithmetic
No common Ratio
Common Difference: $6-3=3, 9-6=3, 12-9=3$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$
$$= \frac{n}{2}[2(3) + (n-1)3]$$
$$= \frac{n}{2}[6 + 3n - 3]$$
$$= \frac{n}{2}[3 + 3n]$$

Find the sum

$$\sum_{k=1}^{60} 4k + 2$$

$$\sum_{k=1}^{20} 200(.3)^k$$

Arithmetic

$$a_1 = 4(1) + 2 = 6, a_2 = 10, a_3 = 14$$

Common difference d=4

$$a_{60} = 4(60) + 2 = 242$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{60} = \frac{60}{2}(6 + 242) = 7440$$

Geometric

$$a_1 = 200(.3)^1 = 60, a_2 = 18, a_3 = 5.4$$

Common ratio r=.3

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$$

$$S_n = 60\left(\frac{1 - .3^{20}}{1 - .3}\right) \approx 85.7143$$

Find the indicated term in each sequence

16th term of 2,8,14,20,…

Arithmetic Sequence

$$a_1 = 2, d = 6$$
$$a_n = a_1 + (n-1)d$$
$$a_{16} = 2 + (16-1)8 = 122$$

10th term of $3, \frac{3}{5}, \frac{3}{25}, \ldots$

Geometric Sequence

$$a_1 = 3, r = \frac{1}{5}$$
$$a_n = a_1 r^{n-1}$$
$$a_{10} = 3\left(\frac{1}{5}\right)^{10-1} = \frac{3}{5^9} = \frac{3}{1953125}$$

Find a general formula for the arithmetic sequence
$a_4 = 34$ and $a_{16} = 58$

Remember $a_n = a_1 + (n-1)d$

$$a_4 = a_1 + 3d = 34$$
$$a_{16} = a_1 + 15d = 58$$

Solve for a_1 and d using the elimination method or substitution

$$a_1 + 3d = 34$$
$$a_1 + 15d = 58$$

Using these methods (from chapter 6)

$$a_1 = 28, d = 2$$

General Form is $a_n = 28 + (n-1)2$

$$a_n = 28 + 2n - 2$$
$$a_n = 26 + 2n$$

Determine if the infinite geometric series converges or diverges. If it converges, find the sum
5, 2.5, 1.25,….

The common ratio is .5 ($\frac{2.5}{5} = .5$)
For an infinite geometric series to converge $|r| < 1$
Since $|.5| < 1$, the series converges

Sum of a convergent infinite geometric series

$$S_\infty = \frac{a_1}{1 - r} = \frac{5}{1 - .5} = 10$$

Use the principle of Mathematical Induction to show that the given statement is true for all natural numbers of n.

$$17 + 35 + 53 + \cdots + (18n - 1) = n(9n + 8)$$

Step 1: Show that the statement is true for n=1

$$17 + 35 + 53 + \cdots + (18n - 1) = n(9n + 8)$$
$$(18(1) - 1) = 1(9(1) + 8)$$
$$17 = 17$$

Assume the statement holds for some k. Determine whether it holds for k+1

$$17 + 35 + 53 + \cdots + (18k - 1) = k(9k + 8) \text{ (Based on the assumption)}$$
$$\underbrace{17 + 35 + 53 + \cdots + (18k - 1)}_{k(9k+8)} + (18(k + 1) - 1) = (k + 1)(9(k + 1) + 8)$$
$$k(9k + 8) \qquad +(18(k + 1) - 1) = (k + 1)(9(k + 1) + 8)$$

$$k(9k + 8) + (18k + 18 - 1) = (k + 1)(9k + 9 + 8)$$
$$k(9k + 8) + (18k + 17) = (k + 1)(9k + 17)$$
$$9k^2 + 8k + (18k + 17) = (k + 1)(9k + 17)$$
$$9k^2 + 26k + 17 = (k + 1)(9k + 17)$$
factored
$$(k + 1)(9k + 17) = (k + 1)(9k + 17)$$

The left and right sides are equal. The statement holds for k+1, given that it holds for k. Therefore, by the Principle of Mathematical Induction, it holds for all natural numbers, n.

Expand the expression using the Binomial Theorem.

$$(2x-3)^5$$

Binomial Theorem is $(x+a)^n = \binom{n}{0}x^n + \binom{n}{1}ax^{n-1} + \cdots + \binom{n}{j}a^k x^{n-j} + \cdots \binom{n}{n}a^n$

Remember $\binom{n}{j} = {}_nC_j = \frac{n!}{j!(n-j!)}$

$$\binom{5}{0}(2x)^5 + \binom{5}{1}(-3)(2x)^4 + \binom{5}{2}(-3)^2(2x)^3 + \binom{5}{3}(-3)^3(2x)^2 + \binom{5}{4}(-3)^4(2x)^1 + \binom{5}{5}(-3)^5$$

$$2^5x^5$$

$$1(32x^5) + 5(-3)(16x^4) + 10(9)(8x^3) + 10(-27)(4x^2) + 5(81)(2x) + 1(-243)$$

$$32x^5 - 240x^4 + 720x^3 - 1080x^2 + 810x - 243$$

Find The coefficient of x^7 in the expression $(x+2)^9$

Use $\binom{n}{j} x^{n-j} a^j$

n=7, j=2 (9-j=7) x=x and a=2

$$\binom{9}{2} x^7 (2)^2 = 36x^7(4) = 144x^7$$

The coefficient is 144

A mosaic tile floor is designed in the shape of a trapezoid 40 feet wide at the base and 20 feet wide at the top. The tiles, 12 inches by 12 inches, are to be placed so that each successive row contains two less tiles that the row below. How many tiles are needed?

The tiles in feet are 1 foot by 1 foot. So the first row will have 40 and the last row will have 20

This is an arithmetic sequence with $a_1 = 40, d = -2$ (each row contains 2 less from the previous) $a_n = 20$ (last row)

1) Find the number of rows, n, using $a_n = a_1 + (n-1)d$

$$20 = 40 + (n-1)(-2)$$
$$20 = 40 - 2n + 2$$
$$20 = 42 - 2n$$
$$-22 = -2n$$
$$n = 11$$

2) Find the total number of tiles (sum) using $S_n = \frac{n}{2}(a_1 + a_n)$

$$S_{11} = \frac{11}{2}(40 + 20) = 330\ tiles$$