Rationalizing Denominators

Square roots:

- Multiply the numerator and denominator by the denominator. Then simplify.

Examples

Simplify the following expression: $\frac{6}{\sqrt{7}}$

$\frac{6}{\sqrt{7}} = \frac{6}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$ *Multiply both numerator and denominator by $\sqrt{7}$

$= \frac{6\sqrt{7}}{7}$

Because we cannot simplify any further, $\frac{6\sqrt{7}}{7}$ is our final answer.

Simplify the following expression: $\frac{10}{\sqrt{5}}$

$\frac{10}{\sqrt{5}} = \frac{10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ *Multiply both numerator and denominator by $\sqrt{5}$

$= \frac{10\sqrt{5}}{5}$ *We see that we can simplify this fraction further by dividing the numerator and denominator each by 5

$= 2\sqrt{5}$

Because we cannot simplify any further, $2\sqrt{5}$ is our final answer.

Simplify the following expression: $\frac{\sqrt{6}}{\sqrt{15}}$

$\frac{\sqrt{6}}{\sqrt{15}} = \frac{\sqrt{6}}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}}$ *Multiply both numerator and denominator by $\sqrt{15}$

$= \frac{\sqrt{90}}{15}$ *Now we check to see whether $\sqrt{90}$ can be simplified, which we can do

$= \frac{\sqrt{9}\sqrt{10}}{15}$ *Split up $\sqrt{90}$ as $\sqrt{9}$ and $\sqrt{10}$ to help us simplify

$= \frac{3\sqrt{10}}{15}$ *Simplify $\sqrt{9}$

$= \frac{\sqrt{10}}{5}$ *Simplify the whole numbers (the 3 and 15) by dividing both the numerator and denominator by 3

Because we cannot simplify any further, $\frac{\sqrt{10}}{5}$ is our final answer.

Simplify the following expression: $\frac{x^2}{\sqrt{2x}}$

$\frac{x^2}{\sqrt{2x}} = \frac{x^2}{\sqrt{2x}} \cdot \frac{\sqrt{2x}}{\sqrt{2x}}$ * Multiply both numerator and denominator by $\sqrt{2x}$

$= \frac{x^2\sqrt{2x}}{2x}$ * Now we check to see whether anything can be simplified. In this problem, we can take an x out of the numerator and the denominator

$= \frac{x\sqrt{2x}}{2}$

Because we cannot simplify any further, $\frac{x\sqrt{2x}}{2}$ is our final answer.

Cube roots:

- Multiply the numerator and denominator by a factor that will create a perfect cube in the denominator. Then simplify.

Examples

Simplify the expression: $\frac{4}{\sqrt[3]{6}}$

$\frac{4}{\sqrt[3]{6}} = \frac{4}{\sqrt[3]{6}} \cdot \frac{\sqrt[3]{6^2}}{\sqrt[3]{6^2}}$ *Because we have one cube root of 6 in the denominator, we multiply by 2 more cube root of 6's $(or\ \sqrt[3]{6^2})$ to create a perfect cube in the denominator.

$= \frac{4\sqrt[3]{36}}{6}$ *Under the cube root, we now have $6^2 = 36$, and we look to see if there is more we can simplify

$= \frac{2\sqrt[3]{36}}{3}$ *Divide both the numerator and denominator by 2 so that we can simplify as much as possible

Because we cannot simplify any further, $\frac{2\sqrt[3]{36}}{3}$ is our final answer.

Simplify the expression: $\frac{7}{\sqrt[3]{x^2}}$

$\frac{7}{\sqrt[3]{x^2}} = \frac{7}{\sqrt[3]{x^2}} \cdot \frac{\sqrt[3]{x}}{\sqrt[3]{x}}$ *Because we have an x^2 in our denominator, we need one more x to make it a perfect cube. Thusly, we multiply by $\sqrt[3]{x}$ on both the numerator and the denominator.

$= \frac{7\sqrt[3]{x}}{x}$ *We look to see if we can simplify further. In this case, we cannot.

Because we cannot simplify any further, $\frac{7\sqrt[3]{x}}{x}$ is our final answer.

Simplify the expression: $\frac{\sqrt[3]{21y}}{\sqrt[3]{3xy^2}}$

$\frac{\sqrt[3]{21y}}{\sqrt[3]{3xy^2}} = \frac{\sqrt[3]{21y}}{\sqrt[3]{3xy^2}} \cdot \frac{\sqrt[3]{9x^2y}}{\sqrt[3]{9x^2y}}$ *In our denominator, we have one 3, one x, and two y's. Thusly, we need two 3's ($or\ 3^2, which\ is\ 9$), two x's ($or\ x^2$), and one y so that they all become perfect cubes. This means we will multiply the numerator and denominator each by $\sqrt[3]{9x^2y}$

$= \frac{\sqrt[3]{189x^2y}}{3xy}$ *Now we look to see if we are able to simplify any further. We want to specifically check if $\sqrt[3]{189}$ can be broken down further by taking out a perfect cube, which in this case it can be, using $\sqrt[3]{27}$ and $\sqrt[3]{7}$

$= \frac{\sqrt[3]{27}\sqrt[3]{7x^2y}}{3xy}$ *Split up $\sqrt[3]{189}$ as $\sqrt[3]{27}$ and $\sqrt[3]{7}$

$= \frac{3\sqrt[3]{7x^2y}}{3xy}$ *Simplify $\sqrt[3]{27}$

$= \frac{\sqrt[3]{7x^2y}}{xy}$ *Cancel out the 3's in both the numerator and denominator

Because we cannot simplify any further, $\frac{\sqrt[3]{7x^2y}}{xy}$ is our final answer.

Denominators with two terms

- Multiply the numerator and denominator by the conjugate of the denominator. Make sure to distribute or FOIL the numerator and denominator. Then simplify.

Examples

Simplify the expression: $\frac{5}{3-\sqrt{2}}$

$\frac{5}{3-\sqrt{2}} = \frac{5}{3-\sqrt{2}} \cdot \frac{3+\sqrt{2}}{3+\sqrt{2}}$ *Multiply by the conjugate

$= \frac{15+5\sqrt{2}}{9+3\sqrt{2}-3\sqrt{2}-2}$ *FOIL the denominator, and distribute the numerator

$= \frac{15+5\sqrt{2}}{7}$ *Simplify the denominator

Because we cannot simplify any further, $\frac{15+5\sqrt{2}}{7}$ is our final answer.

Simplify the expression: $\frac{6+\sqrt{10}}{5+\sqrt{6}}$

$\frac{6+\sqrt{10}}{5+\sqrt{6}} = \frac{6+\sqrt{10}}{5+\sqrt{6}} \cdot \frac{5-\sqrt{6}}{5-\sqrt{6}}$ *Multiply by the conjugate

$= \frac{30-6\sqrt{6}+5\sqrt{10}-\sqrt{60}}{25-5\sqrt{6}+5\sqrt{6}-6}$ *FOIL in both the numerator and denominator

$= \frac{30-6\sqrt{6}+5\sqrt{10}-2\sqrt{15}}{19}$ *Simplify in both the numerator and the denominator. $\sqrt{60} = 2\sqrt{15}$ in the numerator, and the $\sqrt{6}$ terms cancel out in the denominator.

Because we cannot simplify any further, $\frac{30-6\sqrt{6}+5\sqrt{10}-2\sqrt{15}}{19}$ is our final answer.

Simplify the expression: $\frac{1+\sqrt{3}}{1-\sqrt{3}}$

$\frac{1+\sqrt{3}}{1-\sqrt{3}} = \frac{1+\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$ *Multiply by the conjugate

$= \frac{1+\sqrt{3}+\sqrt{3}+3}{1+\sqrt{3}-\sqrt{3}-3}$ *FOIL in both the numerator and the denominator

$= \frac{4+2\sqrt{3}}{-2}$ *Combine like terms

$= -2-\sqrt{3}$ *Simplify further by dividing out a common factor. In this case, we divide by -2, so that we no longer have a denominator (or in other words, our denominator is 1)

Because we cannot simplify any further, $-2-\sqrt{3}$ is our final answer.