Transformations Rules

aka Translation Rules

$f(x) + a$ is $f(x)$ shifted upward a units Ex. Shift 3 units up Before: $y = x^2 + x$, Point (x,y) After: $y = x^2 + x + 3$, Point (x,y+3)	**$f(x) - a$ is $f(x)$ shifted downward a units** Ex. Shift 3 units down Before: $y = x^2 + x$, Point (x,y) After: $y = x^2 + x - 3$, Point (x,y-3)
$f(x + a)$ is $f(x)$ shifted left a units Ex. Shift 3 left Before: $y = x^2 + x$, Point (x,y) After: $y = (x + 3)^2 + (x + 3)$, Point (x-3,y)	**$f(x - a)$ is $f(x)$ shifted right a units** Ex. Shift 3 right Before: $y = x^2 + x$, Point (x,y) After: $y = (x - 3)^2 + (x - 3)$, Point (x+3,y)
$-f(x)$ is $f(x)$ flipped upside down ("reflected about the x-axis") Ex. Reflect over x-axis Before: $y = x^2 + x$, Point (x,y) After: $y = -x^2 - x$, Point (x,-y)	**$f(-x)$ is the mirror of $f(x)$ ("reflected about the y-axis")** Ex. Reflect over y-axis Before: $y = x^2 + x$, Point (x,y) After: $y = (-x)^2 + (-x)$, Point (-x,y)
***cf(x)* where c>1 vertically stretched point (x,y) becomes point(x,cy)** Ex. Vertical Stretch by factor 3 Before: $y = x^2 + x$, Point (x,y) After: $y = 3x^2 + 3x$, Point (x,3y)	***cf(x)* where 0< c<1 vertically shrunk (x,y) becomes point(x,cy)** Ex. Vertical shrink by factor 1/3 Before: $y = x^2 + x$, Point (x,y) After: $y = \frac{1}{3}x^2 + \frac{1}{3}x$, Point (x,1/3y)
***f(cx)* where c>1 horizontal shrunk point (x,y) becomes point $(\frac{x}{c}, y)$** Ex. Horizontal shrink by factor 3 Before: $y = x^2 + x$, Point (x,y) After: $y = (3x)^2 + (3x)$, Point (1/3x, y)	***f(cx)* where 0<c<1 horizontal stretch point (x,y) becomes point $(\frac{x}{c}, y)$** Ex. Horizontal stretch by factor 1/3 Before: $y = x^2 + x$, Point (x,y) After: $y = (\frac{1}{3}x)^2 + (\frac{1}{3}x)$, Point $(\frac{1}{1/3}x, y)$ or Point $(3x, y)$

Tips:

- When the transformation is happening to the x, we write the transformation in parenthesis
- Transformations inside the parenthesis does the inverses

    Ex. $y=(x+3)^2$ move $y=x^2$ in the negative direction (i.e.-3)

    Ex. $y=3x^2$ will not stretch $y=x^2$ by a multiple of 3 , but stretch it by a factor of 1/3
- When the transformation is happening to the y, we write it outside any parenthesis

Helpful Links:

http://www.purplemath.com/modules/fcntrans.htm

http://coolmath.com/algebra/21-advanced-graphing/02-shifting-reflecting-etc-01.htm

http://www.themathpage.com/aPreCalc/translation.htm

http://www.onlinemathlearning.com/precalculus.html (go to section called Functions and Transformation of Graphs Videos)

http://tutorial.math.lamar.edu/Classes/Alg/Transformations.aspx